Chapter 3
How Do We Make Measurements?

Chemistry, like all sciences, has its quantitative side. In this chapter, you will be introduced to the units and types of measurement in chemistry.

3.1 Background

Since the earliest times of human civilization, we have needed measurements. Our measurement system of time dates back many millennia and was devised by the Babylonians who were the early inhabitants of the area now called Iraq. About two thousand years ago, the Romans needed a measurement of how far their armies marched. They took one thousand double-paces (mille passum, in Latin) as their base distance – what we now refer to as a mile. For a much smaller distance, the traditional British (Imperial) measure was an inch, taken as the length of the first joint of the thumb.

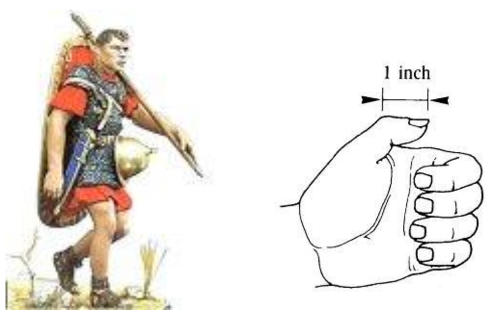


Figure 3.1 (left) the Imperial unit of a mile originates from 1000 double-paces of a Roman soldier (right) the Imperial unit of an inch derives from the length of the first joint of the thumb.

3.2 The International System of Units (SI)

The Imperial system of units, as described above, was not really a system, but a collection of ways of measuring things easily. It was the French scientists at the time of the French Revolution who decided that measurements needed to be systematized and defined in scientific terms. The metric system that they devised in 1875 was the predecessor of the modern International System of Units.

Figure 3.2 A French postage stamp commemorating the 100^{th} anniversary of the adoption of the metric system.

Scientists can express the measurement of any scientific property in terms of seven units or combinations of these units. These units, together with their corresponding names and symbols, are given in Table 1. In this course, it is length, mass, temperature, and amount of substance, which will be of regular use.

Table 3.1 The SI base units and their symbols

Base Unit	Name	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
temperature	Kelvin	K
amount of substance	mole	mol
electric current	Ampère	A
luminous intensity	candela	cd

3.3 Accuracy and Precision

In everyday life, we tend to use the terms 'accuracy' and 'precision' interchangeably. However, in chemistry each word has an exact meaning. When chemists make measurements, such as the levels of environmental pollution, the values reported need to be both accurate and precise.

A good example is personal bathroom weigh scales. Suppose someone stands on the scales and it reads 55 kg. Is that the true weight? First, one has to know if the scales are working properly. Perhaps the scales have an error and always read, say, 5 kg too low. In that case, the scales are not accurate. But to know if the reading is precise, we need to check the increments that the scale measures. Does it measure only in multiples of 5 kg? – In which case, it is not at all precise. Does it measure changes of 1 kg? Or better still, changes of 0.1 kg? In which case the scales are very precise.

The scientific definitions are as follows:
Accuracy: how close the average of a set of measurements comes to the true value
Precision: how closely the values agree among themselves in a set of measurements; or the number of significant figures of a single number.

A commonly used example to differentiate accuracy and precision is the dartboard (Figure 3.4). From this series, you can clearly see the difference in meaning between the two terms.

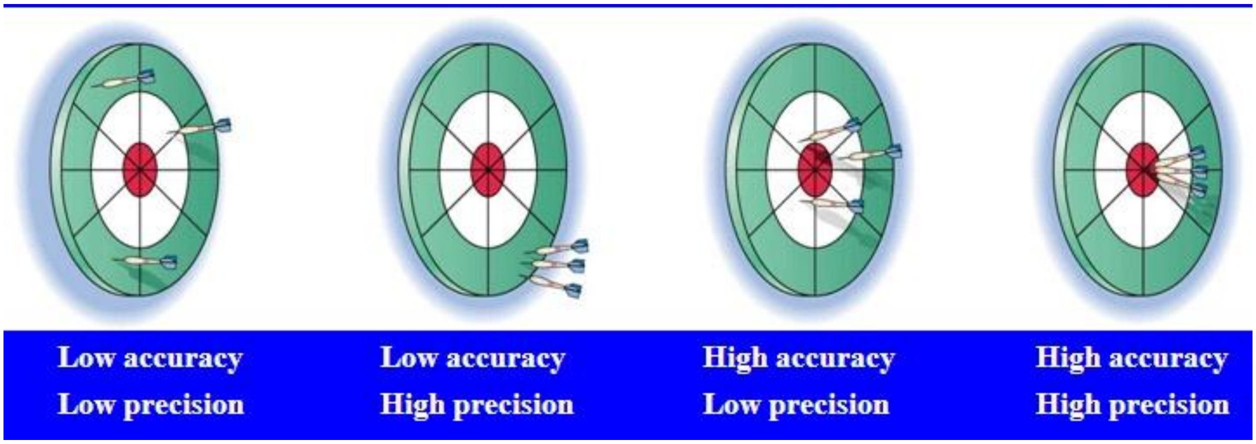


Figure 3.4 The difference between precision and accuracy

To report the degree of accuracy, we use the term error. There are two types of error:
Systematic error – a permanent bias in the results, so that all measured values are consistently higher or lower than the true value.
Random error – a statistical variation in the values, often the fluctuation of reading close to some mean value.

Sometimes a value is misread. We do not want to include the value in our results so it is permissible to exclude the value from our calculations. For example, suppose four students measure the volume of a liquid and report the following values: 3.6 mL, 3.4 mL, 4.3 mL, and 3.8 mL. Clearly, the third student mis-read the scale and it would introduce an error if we included her/his value in our average. So instead, we write:

Measured volumes = 3.6 mL, 3.4 mL. ~~4.3 mL~~, 3.8 mL

Average volume = (3.6 mL + 3.4 mL + 3.8 mL)/3 = 3.6 mL

Assuming we have random errors, averaging three separate measurements will increase the accuracy. We have not increased the precision, as we measured the volume to the nearest $\frac{1}{10}$th of a milliliter, thus (as will be discussed below), our average can only be reported to the nearest $\frac{1}{10}$th of a milliliter.

3.4 Mass Measurement in the Laboratory

In the chemistry laboratory, the mass of an object is obtained using a balance. There are two types that are used, depending upon the precision of the measurement needed, the general-purpose balance and the analytical balance.

Figure 3.5 Two of the types of laboratory balance: an analytical balance (left) and a general-purpose balance (right)

The general-purpose balance is low-cost and typically measures to the nearest 0.1 g or 0.01 g. The more-expensive analytical balance typically measures to the nearest 0.0001 g, that is, the analytical balance is much more precise than the general-purpose balance.

3.5 Significant Figures

As you can see above, using the analytical balance results in more digits than the general-purpose balance. This usually means that the value obtained on the analytical balance is more precise than that on the general-purpose balance. To quantitatively express how precise a number is measured, the term ***significant figures*** is used. When counting the number of significant figures in a number containing a decimal point:

All non-zero digits are counted
Leading zeros, those that precede the first non-zero are not counted
Captive zeros, those between non-zero digits are counted
Trailing zeros, those that follow non-zero digits are counted

In scientific measurement, the decimal point helps define the precision of the measurement. A difficult arises with imprecise numbers. For example, suppose a friend tell you that they have "a hundred dollars." Do they mean somewhere about $100, maybe $95.23 or $102.99; or do they mean they possess a one-hundred dollar bill. To distinguish, the decimal point can be used. Thus a number of 100. would indicate that the value is 100, not 99 or 101. However, the best solution is to convert the number to scientific notation.

3.6 Scientific Notation

In science, numbers can be very large or very small. Instead of writing lots of zeros, it is better to write the number in scientific notation, that is, to 'take out' the powers of ten from the number. In scientific notation, only one digit preceded the decimal point, and the powers of ten are expressed as an exponent. For example, the number 200.0 would be written as 2.000×10^2, while the number 0.0040 would be written as 4.0×10^{-3}. Notice that in each case it was important to keep the same number of significant figures as in the original number. Alternatively, prefixes to the units can be used. As an example, a mass of 0.0027 g can be expressed in scientific notation as 2.7×10^{-3} g or as 2.7 mg (milligrams) as there are 1000 (10^3) milligrams in 1 gram . There are several prefixes that are commonly used and these are listed in the Table below. The prefixes corresponding to positive exponents are designated to have upper-case symbols, and those corresponding to negative exponents, lower-case symbols. However, lower-case 'k' was used for kilo- before the International System was developed, so it has been an exception to the rule.

Table 3.2 Common prefix sizes and names

Prefix size	Prefix name	Prefix
10^9	giga-	G
10^6	mega-	M
10^3	kilo-	k
-	-	-
10^{-1}	deci-	d
10^{-2}	centi-	c
10^{-3}	milli-	m
10^{-6}	micro-	μ
10^{-9}	nano-	n
10^{-12}	pico-	p

CALCULATORS AND CALCULATIONS

Unfortunately, pocket calculators are not designed to make scientific calculations simple. Each calculator type has its own keyboard design, but most have two keys that are vital for chemical calculations. These are shown below:

The exponent (EXP) key represents the '10' part of the power of ten. The (+/–) key makes it possible to change from a positive to a negative exponent. So, for example, to enter 2.7×10^{-3} into the calculator, one would press the '2' then '.' then '7' to enter the number itself, followed by 'EXP' then '+/-' then '3.'

3.7 Significant Figures in Calculations

After scientific data has been collected, it needs to be analyzed, such as averaging a set of values. Suppose we have to find the average of three mass measurements: 8.2 g, 8.4 g, and 8.7 g. Adding the numbers and dividing by three gives a calculator answer of 8.433333333 g. Each of the original measurements was made to the nearest $\frac{1}{10}$th of a gram, yet the average implies that the measurement was made to the nearest 0.000000001 g! Such precision would be beyond the capability of any chemical balance. For this reason, we need a set of significant figure rules when doing scientific calculations.

Significant Figure Rules for Multiplication and Division

There is one rule if the calculation involves multiplying or dividing numbers:
When multiplying or dividing measured values, the answer must have the same number of significant figures as the measurement with the least number of significant figures.

If the answer has more digits than are permissible by the significant figure rules, then the non-significant figures must be eliminated according to the following rounding-off rules:

1. If the first non-significant digit is less than 5, it is dropped and the last significant digit remains the same.
2. If the first non-significant digit is 5 or greater, it is dropped and the last significant digit is increased by one.

Thus calculating 3.6×2.42 gives a calculator answer of 8.712. The two starting numbers have two and three significant figures respectively, so the answer can only have the lesser, that is, two. The correct answer, then, will be 8.7.

Calculators and Significant Figures

Above, it was commented how calculators do not know about significant figures and that they will often produce too many digits. Calculators can also produce too few digits, for they do not recognize trailing zeros. For example, the calculation of 2.0×2.0 will give a calculator answer of 4. Yet both numbers were to two significant figures, so the answer should be to two significant figures. Thus a zero has to be added after the decimal point to give 4.0 as the correct answer.

Significant Figure Rules for Addition and Subtraction

The rules for addition and subtraction are worded differently.
When adding or subtracting measured values, the answer must be given to the same number of places after the decimal point as the measurement that has the least number of places after the decimal point.

WEIGHING BY DIFFERENCE

The addition and subtraction rules are particularly important in the chemical laboratory. Chemists do not pour substances onto the weighing pan, instead, a container is used. The weight (mass) of the substance is found by subtracting the mass of the empty container from the mass of the container plus substance. The same principle is used when weighing gravel or soil carried in dump trucks (see Figure 3.6).

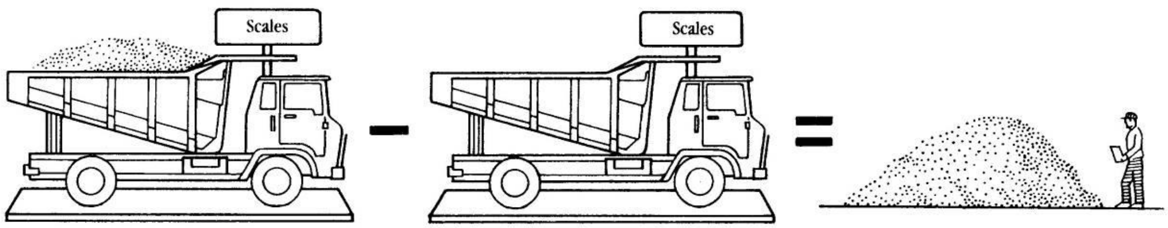


Figure 3.6 The mass of gravel or soil carried in a dump truck is obtained by weighing by difference.

With the addition and subtraction rules, the answer can have a different number of significant figures to the starting numbers. For example, suppose the mass of an empty container is 3.04 g and the measured mass of the container with a substance is 3.98 g. To do the subtraction, it is wisest to 'stack' the numbers:

Mass of container plus substance =	3.98 g
Mass of empty container =	3.04 g
Mass of substance =	0.94 g

Both of the masses were measured to the nearest $\frac{1}{100}$th of a gram (two places after the decimal), so the answer will be reported to the nearest $\frac{1}{100}$th of a gram. As a result, there are only two significant figures in the answer, even though both measurements are precise to three significant figures. Just as it is possible to lose a significant figure in a subtraction, so it is possible to gain a significant figure during an addition.

EXACT NUMBERS

Exact numbers are excluded when counting significant figures. For example, there is exactly 1000 m in 1 km. Any calculation including these values would disregard them for significant figure purposes.

3.8 The Conversion-Factor Method

In this text, the conversion-factor method (also called dimensional analysis) will be used to solve chemical calculations. Here the terminology of the method will be introduced together with some simple everyday examples to illustrate how the method works.

The Unit Factor – this is a ratio that quantifies the relationship between one unit and another;

The Strategy – the pathway from the quantity given to the quantity desired;
The Relationship – the needed unit factors written as equivalents.

THE ALGEBRA OF UNITS

Of equal importance to the numbers themselves, it is crucial to express the result in the correct units. Units are manipulated in the same way as numbers. Here are three examples:

$\text{cm}^2 + \text{cm}^2 = \text{cm}^2$

$\text{cm} \times \text{cm}^2 = \text{cm}^3$

$\text{g} \div \text{cm}^3 = \text{g}\cdot\text{cm}^{-3}$

The methodology for solving problems is shown in the following example.

EXAMPLE 3.1

Convert 0.064 m to mm.

Answer

First, planning the strategy. In this case, the calculation can be performed in one step: a change in length units, m → mm.

Second, identifying the unit factor. In this case, the prefix ‘m’ represents a factor of 1000, so the relationship is that 1 m ≡ 1000 mm (where ‘≡’ means ‘equivalent to’).

Third, the calculation can now be solved by writing the name of what we have to find, then the numerical value given, and multiply by the unit factor as a ratio.

$$\text{Length(mm)} = 0.064\ \text{m} \times \left(\frac{1000\ \text{mm}}{1\ \text{m}}\right) = 64\ \text{mm}$$

The units of ‘m’ cancel, giving the units of ‘mm.’ Note that the given number was to two significant figures and the ratio contained exact numbers, so the answer must be expressed to two significant figures.

EXAMPLE 3.2

Convert 425 mg to g.

Answer

In this example, the strategy and relationship will be displayed in list form, as will be done in all subsequent examples.

<u>Strategy</u>	<u>Relationship</u>
mg → g	1 g ≡ 1000 mg

$$\text{Mass(g)} = 425\ \text{mg} \times \left(\frac{1\ \text{g}}{1000\ \text{mg}}\right) = 0.425\ \text{g}$$

3.9 Volume

Volume is a derived unit, not a base unit. A derived unit is one that contains more than a single base unit, in this case, the unit of length cubed, or mathematically m^3. A cubic metre is a very large volume. In practical terms, smaller volume units are necessary. A $\frac{1}{10}$th of a metre is a decimeter, that is 1 m ≡ 10 dm, thus for volume we need the cubed equivalence:

$$(1 \text{ m})^3 \equiv (10 \text{ dm})^3 \equiv 1000 \text{ dm}^3$$

In other words, there are 1000 dm^3 in one m^3

Even a cubic decimeter is quite large, so a smaller unit is most common, that is, the cubic centimeter. A centimetre is $\frac{1}{10}$th of a decimeter and $\frac{1}{100}$th of a metre. The cubic equivalence can be found the same way as above.

$$(1 \text{ dm})^3 \equiv (10 \text{ cm})^3 \equiv 1000 \text{ cm}^3$$

In other words, there are 1000 cm^3 in one dm^3.

When measuring liquids, it is more convenient to report volumes in terms of litres (symbol L). Table 3.3 shows the equivalencies.

Table 3.3 Volume equivalents for solids and liquids

solid volume	**liquid volume**
m^3	kL
dm^3	L
cm^3	mL

Figure 3.7 shows the relationships between the common volume units.

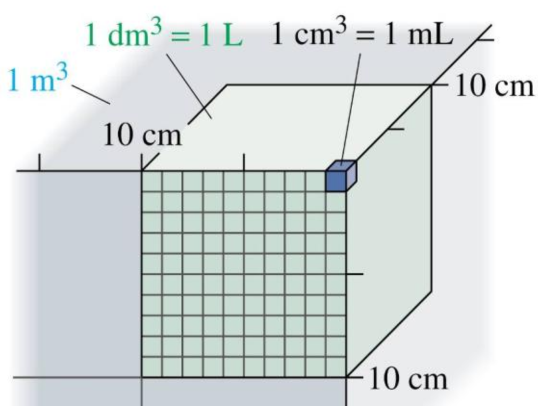


Figure 3.7 A comparison of the common volume units

In the chemical laboratory, there are many named items of glassware. In Figure 3.8, three types are shown: the beaker for holding solutions of chemicals; the graduated cylinder for dispensing approximate volumes; and the buret (also spelled burette) for dispensing precise volumes.

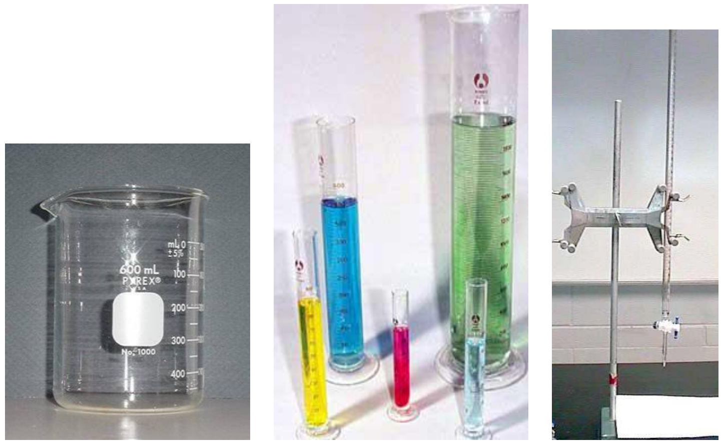


Figure 3.8 from left to right: a beaker; different sizes of graduated cylinders; and a buret.

Each of these items of glassware enable the volume to be read to a different degree of precision. Figure 3.9 shows some examples.

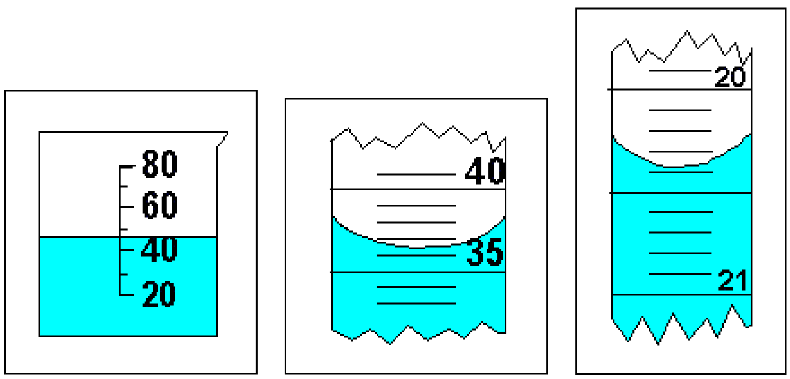


Figure 3.9 Examples of precision for common items of glassware. From left to right, a beaker, a graduated cylinder, and a buret.

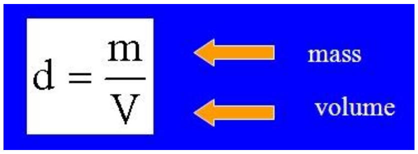
For the beaker, the long lines are 20 mL apart and the shorter lines are between at 10 mL. The top of the liquid layer, called the meniscus, can be read as somewhere between 40 mL and 50 mL. It is possible to obtain a more precise value by ***interpolation***. To interpolate, you must imagine smaller divisions than what are on the scale. In this case, imagine a gradation at 45 mL, half-way between 40 mL and 50 mL. The meniscus, then, would be just above that imaginary 45 mL mark. So the volume would be estimated as about 46 mL or 47 mL or 48 mL.

For the graduated cylinder, each long line is at every 5 mL and each short line at the intervening 1 mL intervals. It is always the bottom of the meniscus that is read and, in this case, the volume must be somewhere between 36 mL and 37 mL. Again it is possible to interpolate. The bottom of the meniscus is close to half-way between 36 mL and 37 mL, so the volume is about 36.5 mL. The volume can be read more precisely with a graduated, the precision being about ±0.5 mL.

For the buret, note that the scale goes in the reverse direction with the larger number below the small. This is because a buret has a tap in the bottom and the buret is used for dispensing very precise volumes of liquid. Here, the long lines are every 0.5 mL and the short lines at each 0.1 mL interval. Counting downwards, the bottom of the meniscus is below the 20.3 mL line but not quite at the 20.4 mL line. Thus the volume reading must be about 20.37 mL or 20.38 mL (either would be acceptable). The volume can be read much more precisely using a buret, the precision being about ±0.02 mL.

3.10 Density

Another derived unit is density. In Chapter 2, density was mentioned as a measure of the compactness of matter. Having introduced the technique of calculations, here it will be discussed in a quantitative sense. Density is defined mathematically as:



Usually, the mass is measured in grams and the volume in cubic centimetres (or millilitres for liquids). Table 3.4 provides some examples of densities of common substances.

Table 3.4 Densities of some common substances

Substance	**Phase**	**Density ($g{\cdot}cm^{-3}$)**
lead	solid	11.4
iron	solid	5.9
aluminum	solid	2.7
water	liquid	1.0
alcohol	liquid	0.8
air	gas	0.0012
helium	gas	0.0002

Thus gases have very low densities; liquids have densities about 1 $g{\cdot}cm^{-3}$; and solids significantly greater. Density is very important in many contexts. For example, aircraft are made of aluminum because it is a low-density metal. That way, the plane will weigh much less than if it was constructed of iron.

Measuring the Density of a Liquid

To find the density of a liquid, first weigh an empty graduated cylinder, then place a volume of liquid in the cylinder. Re-weigh the cylinder plus liquid. The difference will give the mass of the liquid. Then read the volume of the liquid. Having measured both the volume and the mass, the density can be calculated.

EXAMPLE 3.3

The mass of an empty 100 mL graduated cylinder is 54.86 g. A liquid is poured into the cylinder, giving a total mass of 86.72 g. The volume of the liquid is recorded as 41.5 mL. Calculate the density of the liquid.

Answer

First, we need to find the mass of the liquid.

Mass of cylinder plus liquid	=	86.94 g
Mass of empty cylinder	=	<u>54.86 g</u>
Mass of liquid	=	32.08 g

$$\text{density} = \left(\frac{\text{mass}}{\text{volume}}\right) = \left(\frac{32.08\ \text{g}}{41.5\ \text{mL}}\right) = 0.773\ \text{g} \cdot \text{mL}^{-1}$$

Note that the answer is only to three significant figures as the volume is only measured to three significant figures. As 1 mL ≡ 1 cm^3, the answer could also be expressed as 0.773 g·cm^{-3}.

MEASURING THE DENSITY OF A SOLID

To measure the density of a solid, it is necessary to find a way to determine the volume of the solid. If the solid is a cube or a cylinder, the dimensions could be measured and the volume calculated using algebra. Most solid samples, such as minerals, are irregular in shape. So an indirect method has to be used. If a graduated cylinder is part filled with a liquid, the volume measured, then the solid object is immersed in the liquid, the difference between that volume and the intial volume will give the volume of the solid. The density can then be calculated as before.

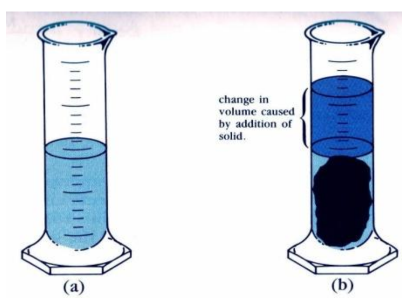


Figure 3.10 To measure the volume of an irregular solid, (a) part-fill a graduated cylinder with a liquid and measure the volume, then (b) insert the solid, the difference in readings will give the volume of the solid.

EXAMPLE 3.5

The mass of a metallic object, claimed to be gold, is 277.30 g. Water is poured into a 100 mL graduated cylinder until the volume reading is 41.5 mL. The lump of metal is carefully inserted into the cylinder to give a new volume reading of 88.5 mL. What is the density of the metal? Is it gold (density 19.3 $g \cdot cm^{-3}$)?

First, we need to find the volume of the solid.

Volume of liquid plus object	=	88.5 mL
Volume of liquid	=	<u>41.5 mL</u>
Volume of object	=	47.0 mL

$$\text{density} = \left(\frac{\text{mass}}{\text{volume}}\right) = \left(\frac{277.30\ \text{g}}{47.0\ \text{mL}}\right) = 5.90\ \text{g} \cdot \text{mL}^{-1}$$

The density of the metal is 5.90 $g \cdot cm^{-3}$ and so the object is definitely not gold! In fact, the value is the same as iron, so it is probably a lump of iron coated in gold paint.

3.11 Where Next?

Having introduced the problem-solving technique of strategy and relationships, in the next Chapter, they will be applied to solving simple chemical calculations.